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Temperature profiles are calculated in tissue models exposed to microwave radiation, by solving the heat transport equation taking into account thermal conduction, thermal convection due to blood flow, and surface cooling of the tissue. We consider two idealized models representing limiting cases of the microwave heating behavior of actual tissue sections. The first consists of a semi-infinite tissue slab exposed to plane electromagnetic radiation; the second considers focused microwave heating in a finite volume of tissue. In both cases, the steady state temperature profile is largely determined by the blood flow and can be considerably different from the microwave energy deposition pattern. In the steady state, the blood flow at physiological levels creates an effective diffusion length of about 1 cm which averages out the temperature variation in the tissue.

Introduction

When tissue is exposed to microwave radiation, it absorbs some of the energy and is heated. Previous studies (1,2,3) have estimated the transient temperature increase with time in a semi-infinite slab of tissue, irradiated by plane microwave radiation. Because the cooling effects of blood flow and heat loss from the tissue surface were neglected in these studies, the calculated tissue temperature rose without limit, and no steady state solution existed. We now consider these effects, and calculate the steady state temperature distribution which results. While the transient solution to the heat transport equation cannot be given here for reasons of space, we estimate by dimensional analysis the time required for the tissue temperature to stabilize after the microwave field is applied.

Although these calculations assume an oversimplified thermal model for tissue, they are useful for two reasons. First, they demonstrate the large effect blood flow has on the microwave-induced heating pattern. Thus, microwave irradiation of tissue and of "phantom" models with the same shape and dielectric properties as tissue results in greatly different steady state heating patterns. Second, they show the limited extent to which the tissue heating pattern can be experimentally controlled by artificially cooling the skin, in an attempt, for example, to "focus" the microwave heating in deeper levels in the tissue for therapeutic applications.

Heat Transport in Tissues

The heat transport equation in tissue is (4)

$$\rho c \frac{\partial T'}{\partial t} = k \nabla^2 T' - V_s (T' - T_o) + Q(x,t) \quad (1)$$

where

ρ = the density of tissue in g/cm^3

c = the specific heat of tissue plus blood in $\text{cal/g}^\circ\text{C}$

k = the coefficient of heat conduction in $\text{cal/cm sec}^\circ\text{C}$

V_s = the product flow and heat capacity of blood in $\text{cal/cm}^3 \text{ sec}^\circ\text{C}$

$Q(x,t)$ = the heat input due to the EM fields in $\text{cal/cm}^3 \text{ sec}$

$T(x,t)$ = the tissue temperature in $^\circ\text{C}$

T_o = the temperature of the arterial blood entering the tissue.

The term $V_s(T' - T_o)$ represents the contribution of the blood perfusion to the dissipation of heat deposited by the microwave field. Wulff(5) has shown that this term should be refined to include the three dimensional nature of the blood flow. Our simpler expression however, produces reasonable temperature profiles in other theoretical models (4,6,7) and is retained here.

Typical values for the thermal constants of tissue are:

k = .0001 cal/cm sec $^\circ\text{C}$

ρc = 1 cal/ cm^3 $^\circ\text{C}$

V_s = 0.0013 cal/ cm^3 sec $^\circ\text{C}$ (average blood flow in man)

We now solve Eq (1) for two special cases mentioned above.

Temperature Rise in Hot Spots

It is well known that microwave-induced heating of finite tissue volumes can become extremely nonuniform, because of the presence of standing waves in the object. Previous work(8) has demonstrated that when 1 GHz plane waves illuminate a tissue sphere of radius 5 cm, a "hot spot" of radius ~ 1 cm occur at its center. An incident power density of 10 mW/cm^2 results in the deposition of $\sim 10 \text{ mW/cm}^3$ (or $Q = 2.4 \times 10^{-3} \text{ cal/cm}^3 \text{ sec}$) in the center of the sphere. We approximate the actual complicated heating pattern by considering a uniformly heated tissue sphere of radius A (1 cm) embedded in a much larger volume of unheated tissue. Taking into account the boundary conditions which require continuity of temperature and heat flow at the edge of the heated region, the steady state solution is:

$$T_1 = \frac{q}{\lambda} \left[1 - (\rho+1) e^{-\rho} \frac{\sinh R\sqrt{\lambda}}{R\sqrt{\lambda}} \right] + T_o, \quad R < A \quad (2)$$

$$T_2 = \frac{q}{\lambda} \left[(\rho-1) e^\rho + (\rho+1) e^{-\rho} \right] \frac{e^{-R\sqrt{\lambda}}}{\sqrt{\lambda R}} + T_o, \quad R > A \quad (3)$$

where

$$\rho = A\sqrt{\lambda}, \quad \lambda = \frac{V_s}{k} \quad \text{and} \quad q = \frac{Q}{k}. \quad (4)$$

The microwave induced temperature profile appears in Figure 1a. In the same figure the effect of the blood flow is also shown. The maximum temperature (which appears at the center of the hot spot) is given by

$$T_{\max} = \frac{q}{\lambda} [1 - (A\sqrt{\lambda} + 1)e^{-A\sqrt{\lambda}}] + T_0 \quad (5)$$

This maximum induced temperature as a function of the blood flow is shown in Fig. 1b). For no blood flow ($\lambda = 0$) Eq. (5) reduces to

$$T_{\max} = \frac{qA^2}{2} \quad (6)$$

For large λ , i.e., $\lambda > \frac{1}{A^2}$,

$$T_{\max} = \frac{q}{\lambda} \quad (7)$$

Note that most of the temperature reduction due to blood flow takes place at physiological values of the blood flow parameter, and that a moderate additional increase of flow (due, for example, to vasodilation) does not greatly reduce the differential temperature rise.

Temperature Rise in a Semiinfinite Tissue Slab

Consider now a plane wave incident upon a semi-infinite slab of tissue. This is an idealization of a typical clinical application of microwave tissue heating, where the treated tissue section is much larger than the wavelength of the radiation in the tissue. The energy input is, for a plane EM wave of intensity I_0 watts/cm² incident upon the tissue,

$$Q(x,t) = \frac{I_0 \tau}{JL} \exp(-x/L) \quad (8)$$

where J is the mechanical equivalent of heat, L is the energy penetration depth for the radiation, and τ is a fraction of energy transmitted into the tissue (the remaining energy being reflected). For muscle tissue, τ is near 0.4 at 2.4 GHz (8). The microwave energy penetration depth L varies in muscle from 1.0 cm to 0.1 cm, at 2.4 and 10 GHz, respectively (8).

The boundary conditions are:

$$T(\infty, t) = T_0, T \geq 0 \quad (9)$$

$$\frac{\partial T}{\partial x}(0, t) = \alpha(T - T_e), t \geq 0 \quad (10)$$

where T_e is the temperature outside of the tissue plane. The last equation describes the heat loss from the tissue, the coefficient α varying widely with environmental conditions. Wissler (4) quotes a value of 0.25 cm⁻¹ for α for a nude human resting in a 30°C environment, increasing by a factor of five or so as the skin is wetted. If the surface of the tissue is artificially maintained at T_e , $\alpha \rightarrow \infty$.

The steady state solution to this equation consistent with the given boundary condition is:

$$T(x, \infty) = \frac{q_0}{\lambda - 1/L^2} \left[\exp(-x/L) - \frac{1/L + \alpha}{\sqrt{\lambda} + \alpha} \exp(-x/\lambda) \right] + \frac{\alpha(T_e - T_0)}{\alpha + \sqrt{\lambda}} \exp(-x/\lambda) + T_0, \quad (11)$$

where parameters q and λ are defined in Eq. (4). This steady state temperature distribution for two penetration depths as shown in Fig. 2a. Note that, because of the blood flow, the effective heating depth can be much larger than the energy penetration depth in the tissue, an effect particularly noticeable for 10 GHz radiation.

This steady state result allows us to estimate how far the maximum tissue temperature increase can be displaced from the surface of the tissue. Figure 2b shows the steady state heating pattern for both an insulated and a well cooled tissue surface, for a total absorbed power of 100 mW/cm² radiation whose penetration depth is appropriate to 2.5 GHz radiation. The position of the temperature maximum is variable by only a cm or so, and less at higher microwave frequencies. Lower frequency radiation (i.e. below 1 GHz) penetrates more deeply into the tissue, but the heating pattern is correspondingly spread out over a much larger tissue volume. Thus the ability to "focus" the tissue heating to a predetermined depth by cooling the tissue surface is limited in this model and probably in clinical situations as well.

The transient solutions for the models considered above are easily obtained but mathematically involved. However, the time required for the tissue temperature to reach the equilibrium after the microwave field is applied is determined by the time constants μ/λ and μL^2 , equal to 750 and 1000 seconds, for the case of the semi-infinite tissue slab exposed to 2.3 GHz radiation. Similar time constants are found for the hot spot model calculation as well.

Conclusions

The two cases considered here represent in effect limiting cases encountered in the microwave heating of tissue, and the heating produced in a clinical application would probably fall within these two extremes. Our results have several implications.

First, microwave irradiation at the generally recognized "safe" level of 10 mW/cm² is not likely to increase tissue temperature more than 0.5°C. Second, because of the effect of blood flow, measurements of the transient temperature increase in microwave-irradiated tissue models might lead to reasonable estimates of the microwave absorption pattern in tissue, but not of the resulting steady-state temperature profile in the tissue. Third, tissue blood perfusion is an important factor determining the steady-state temperature profile in microwave irradiated tissue. In tissue there is an effective thermal diffusion length of 1-2 cm, which leads to an effective averaging of microwave heating patterns over distances of this magnitude. To some extent, this effect will reduce the non-uniform tissue heating due to "hot spots", and also the ability to focus the microwave heating to specific regions in the tissue by external means.

References

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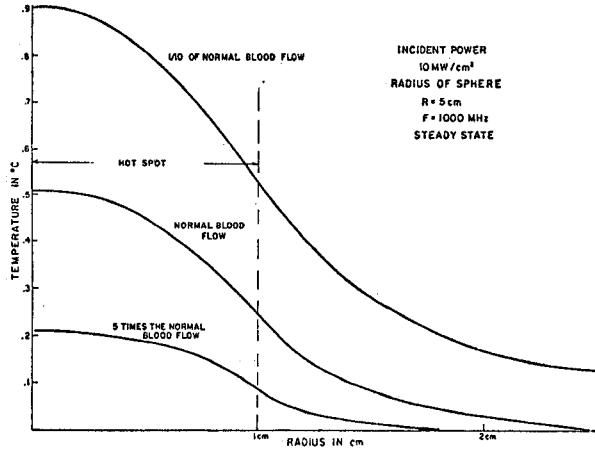


Fig. 1a. Predicted steady state temperature profile resulting from a microwave-induced "hot spot" surrounded by a large region of unheated tissue. The effect of large changes in the assumed value of the blood flow is also shown.

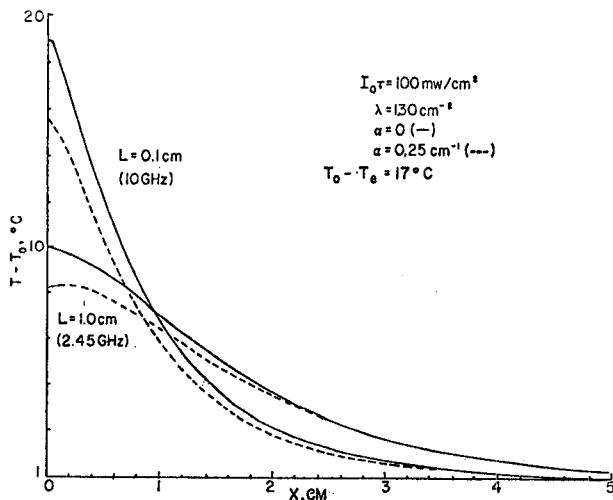


Fig. 2a. The steady state temperature increase a semi-infinite tissue plane produced by microwave irradiation, with energy penetration depths of 0.1 and 1.0 cm.

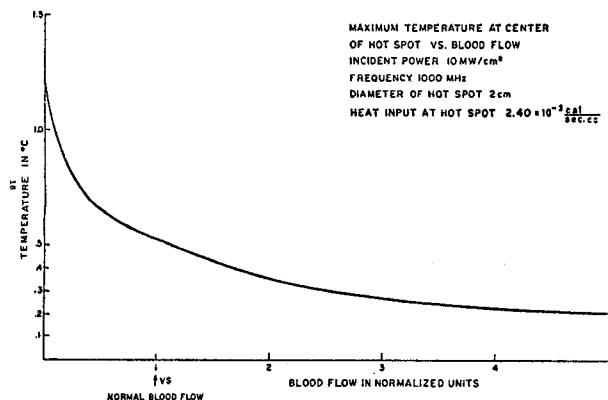


Fig. 1b. The maximum temperature in this hot spot, as a function of blood flow. A moderate increase in blood flow, for example due to vasodilation in the heated tissue, will not greatly change the predicted temperature profile.

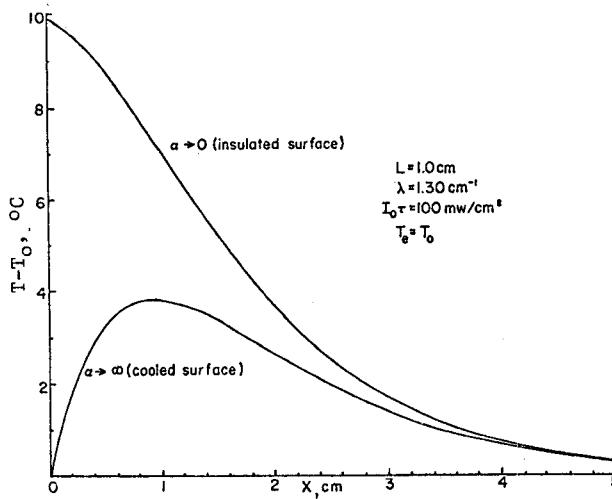


Fig. 2b. Steady state temperature distribution when the tissue surface is insulated (curve A) or cooled to a temperature T_0 (curve B). These functions show how much the microwave heating pattern can be varied by artificially cooling the tissue surface.